The RAND Journal of Economics

ORIGINAL ARTICLE OPEN ACCESS

Strategic Influencers and the Shaping of Beliefs

Akhil Vohra[1,2]

[1]University of Exeter Business School | [2]Terry College of Business, University of Georgia

Correspondence: Akhil Vohra (A.Vohra@exeter.ac.uk)

Received: 5 August 2025 | **Accepted:** 5 August 2025

Keywords: advertising | competition | influencers | networks | targeting

ABSTRACT

Influencers, from propagandists to sellers, expend vast resources targeting agents who amplify their message through word-of-mouth communication. While agents differ in network position, they also differ in their bias: Agents may naturally read articles with a particular slant or buy products from a certain seller. Absent competition, an influencer prefers targeting central agents and those biased *against* it. If agents are unbiased, competition leads to influencers targeting more central agents. However, when agents have heterogeneous biases and competition is intense, the incentive to deter one's rival dominates. Influencers protect their base, targeting those with similar beliefs in equilibrium.

JEL Classification: D83, D85, L10

1 | Introduction

Strategic influencers, ranging from propagandists to sellers, expend vast resources targeting individuals, employing tools such as customized advertisements, sponsored posts, and online recommendations (Bergemann and Bonatti 2011; Fainmesser and Galeotti 2015, 2020). Existing technologies allow them to target recipients at a granular level, increasing direct interaction. Importantly, their message can be amplified by the peer networks of those they target. But whom should they target? Many suggest that it is best to target the most central agents so as to maximize the diffusion of one's message (e.g., Coleman et al. 1966; Galeotti and Goyal 2009; Banerjee et al. 2013, 2019; Beaman et al. 2021). However, a critical feature of the settings used to support these results is that there is a single influencer attempting to "seed" the network, and agents only interact with each other outside of the seeding event. In reality, agents interact repeatedly not only with their peers but also with various external sources, some of whom may be competing with one another. Importantly, agents are often biased and inclined to interact with external sources that reinforce their biases.

To understand why bias matters, consider a social network where users learn about a political event from their peers and from articles they read while browsing the internet. These users may be naturally biased in one direction or the other. That is, when they browse the internet, they will not only see articles from external news sources that specifically target them but also articles from like-minded media. Suppose a left-leaning propagandist targets the users in this social network with the goal of driving the "average" opinion regarding some event toward the left. Although the propagandist will consider a user's centrality in the network, it must also consider the user's bias. Why? Because the marginal gain from targeting a user, say, who already receives persistent impressions from *other* left-leaning sources is much lower than the potential gain from targeting a user who is biased to the right. In other words, targeting users and displacing attention directed toward sources with a similar slant is not as beneficial as displacing attention that would otherwise be directed toward opposing sources. Thus, with limited resources, influencers may need to trade-off between these two features. For example, should a propagandist use funds to reach across the aisle or target her base, and how should she balance this decision with the benefit from targeting central agents? Likewise, sellers can benefit from word-of-mouth communication by targeting influential consumers in the network, but is this irrespective of the consumers' bias?

To understand why competition matters, consider that the left-leaning propagandist's decision to target specific agents will depend significantly on the actions of its rival right-leaning propagandist. When competing for attention, the marginal value of targeting an agent will likely be tied to how much the right-leaning propagandist is investing in that same target. Should the propagandists then pursue the same agents or could that be a waste of resources? Alternatively, should the propagandists segment the market and concentrate their efforts on different agents?

To analyze the effect of bias and competition on influencer targeting decisions, I develop a model of belief formation where agents learn from their neighbors and external sources of information via a simple DeGroot heuristic. These external sources include strategic influencers and non-strategic "private sources". The influencers push their specific beliefs, whereas the non-strategic private sources reinforce an agent's initial belief. Influencers compete over attention to shape the average belief in the network. They have a fixed budget and target agents by spending money to increase the per-period frequency of direct engagement between themselves and the agent. Crucially, the interaction rate between an influencer and an agent depends not only on the influencer's spending but also on her competitor's spending. Such a model is particularly well-suited for analyzing marketing and political competitions. Consumers and voters are typically biased toward a specific firm or ideology, and their beliefs are influenced both by interpersonal communication and targeted external messaging (Huckfeldt et al. 2004; Trusov et al. 2009; Iyengar et al. 2011). Furthermore, firms and political groups spend massive amounts on advertising and data brokers to target advertisements and capture the attention of consumers and voters. In 2021, US organizations spent almost \$22 billion on audience data, to go along with \$221 billion in digital advertising.[1] In the 2024 election cycle, digital advertising spending is expected to reach \$3.5 billion dollars.[2]

I analyze settings with a single influencer and two competing influencers. In the single-influencer setting, the influencer discounts an agent's centrality by their initial, persistent belief. Targeting agents biased in the influencer's favor is not valuable because one is merely displacing attention directed toward private sources already sending similar messages. Thus, the influencer favors agents biased in the opposite direction. In the competitive setting, two influencers engage in a simultaneous move game to drive the average belief in opposite directions. When agents are unbiased, influencer targeting strategies align with those in the single-influencer case: They spend more on central agents. Competition does not distort incentives. However, when agents are biased, competition can alter equilibrium targeting strategies relative to the single-influencer setting. This is because spending on an agent has a two-fold effect in the competitive setting: It increases direct interaction and decreases attention paid toward the competitor. The former is an *acquisition incentive*, whereas the latter is a *deterrence incentive*. Influencers must weigh the combined effect when determining whom to target. As a result, equilibrium targeting strategies are sensitive to how attention is divided based on the influencers' spending. When the deterrence incentive dominates, and spending decreases attention paid to the competitor more than it increases direct interaction for the targeting influencer, influencers prioritize reducing their competitor's influence more than increasing their own. They expend resources to prevent agents from being "turned" by their rivals and target agents biased in their direction. Propagandists will focus on their base, and firms will spend their marketing budget on consumers partial to their products.

In the next section, I describe the model. Section 3 examines single-influencer targeting. Section 4 examines targeting under competition. I defer discussion of the literature to the end.

2 | Model

There are N agents, labeled $\{1, 2, \ldots, N\}$, each holding an initial belief $b_i \in [0, 1]$. Agents are embedded in a *peer network*, a directed graph defined by a non-negative, row stochastic matrix P. P_{ij} represents the frequency with which agent i interacts with j or the relative level of trust agent i places in j.

External to the peer network is a set of *external sources*. These include two strategic influencers and a group of "private sources". The first influencer, M_1, has belief 1, whereas the other, M_2, has belief 0. The influencer's belief is the message it desires to promote. In addition to the influencers, there is a private source S_i, corresponding to each agent i, with belief b_i. As b_i controls agent i's initial belief and the private source's belief, I refer to b_i as the agent's bias.

2.1 | Interaction and Communication

The influencers and private sources constitute *the set of external sources*. Fix a level of external attention $\alpha_i \in [0, 1)$. Each period, agent i interacts with the external sources with probability $\alpha_i \in [0, 1)$. With probability $1 - \alpha_i$, agent i interacts with her peers according to the matrix P. When agent i interacts with the external sources, he receives information from M_1 and his agent-specific private source S_i. To illustrate, consider a setting where M_1 is a liberal propagandist, and M_2 is a conservative propagandist that target a moderate agent who learns outside his peer network through browsing the internet. Although he reads articles from both propagandists, he also receives information from like-minded media (private source S_i) that reinforce his initial belief (bias). Importantly, targeted spending by the influencer affects the interaction rate between an agent and her private source: It *diverts* attention away from i's private sources toward the influencer. The private sources, S_i, are non-strategic with no targeting ability, allowing me to capture a passive persistence of bias.[3]

The probability with which agent i learns from *an external source* is fixed. However, conditional on interacting with external sources, the probability agent i interacts with a given influencer is endogenous. If M_j targets agent i, it can secure some portion of the attention that i gives to external sources. Formally, each influencer has a budget of 1 to allocate across agents in the network. The allocation decision is M_j's targeting strategy. Given a targeting strategy $a^j \in [0, 1]^N$, a competition function f :

$\mathbb{R}^2 \longrightarrow [0,1]$ determines the fraction of α_i that M_j wins. The fraction of α_i an influencer wins, $f(\cdot,\cdot)$, depends on her spending and her competitor's. If agent i interacts with external sources, he interacts with M_j with probability $f(a_i^j, a_i^{-j})$ and from S_i with probability $1 - f(a_i^1, a_i^2) - f(a_i^2, a_i^1)$.[4]

Assumption 1.

1. $f(x, y) + f(y, x) \leq 1$
2. f increasing and concave in its first argument
3. f is decreasing and convex in its second argument

The first condition is technical, ensuring that the combined fraction of α_i won by both influencers does not exceed 1. The second condition is a standard diminishing returns property from additional spending. The third condition is a diminishing returns effect of the *opposition's spending* on one's winnings.[5]

Each external source can be viewed as an additional node in the network that does not update its own belief. Whereas other nodes (i.e., the agents) learn from external sources, each external source only learns from itself. For ease of exposition, define diagonal matrices D^α and D^S, where $D^\alpha_{ii} = 1 - \alpha_i$ and $D^S_{ii} = \alpha_i(1 - f(a_i^1, a_i^2) - f(a_i^2, a_i^1))$. The first two represent interaction rates between agents and external sources, and the third represents the distance of agents' initial beliefs from 1. Communication can then be described via weighted-average updating according to the $(2N+2)\times(2N+2)$ matrix P^*:

$$P^* = \begin{bmatrix} D^\alpha P & \alpha f(a^1, a^2) & \alpha f(a^2, a^1) & D^S \\ \mathbf{0}_{1\times N} & 1 & 0 & \mathbf{0}_{1\times N} \\ \mathbf{0}_{1\times N} & 0 & 1 & \mathbf{0}_{1\times N} \\ \mathbf{0}_{1\times N} & 0 & 0 & \mathbf{I}_{N\times N} \end{bmatrix}$$

The top left block $D^\alpha P$ corresponds to peer-to-peer communication. In an abuse of notation, $\alpha f(a^j, a^{-j})$ denotes the vector of interaction frequency between agents and M_j, with the i^{th} component equal to $\alpha_i f(a_i^j, a_i^{-j})$. D^S corresponds to the direct interaction rates from the fixed private sources. The last three rows of the block matrix correspond to the external sources: each external source places weight 1 on itself.

Agents update their beliefs each period according to a DeGroot heuristic: beliefs at time t are ${P^*}^t b$.[6] The influencers want the average limiting belief in the network to match their own. The average limiting belief in the network is:

$$B(a^1, a^2) = \lim_{t\to\infty} \frac{1}{N} e^T P^* b, \text{ where } e^T \text{ is a row vector of 1's.}$$

Influencer M_1 wishes to maximize $B(a^1, a^2)$, whereas M_2 wishes to minimize it.[7]

It is important to note that the term "belief" need not be interpreted literally. For instance, in a marketing context, where influencers are firms, beliefs could reflect the probability that a consumer views one firm as better than another, capturing brand reputation. In a political context, where influencers are propagandists or political groups, beliefs might reflect a left- or right-wing view about a state of the world. Importantly, these beliefs can be linked to actions. A consumer's belief about the superior firm can translate into their purchase frequency. A voter's beliefs can represent the likelihood of voting for a particular party. More generally, "beliefs" can also be viewed as behaviors that individuals adjust based on interactions with their peers and external sources.

3 | Optimal Targeting: Single-Influencer

3.1 | Targeting Strategy

I begin by considering a setting with a single strategic influencer, M_1. To "convert" the model to this setting, I simply eliminate the second influencer and use a single-variable competition function f. Communication and learning can then be described via weighted-average updating according to the $(2N+1)\times(2N+1)$ matrix P^*:

$$P^* = \begin{bmatrix} D^\alpha P & \alpha f(a^1) & D^S \\ \mathbf{0}_{1\times N} & 1 & \mathbf{0}_{1\times N} \\ \mathbf{0}_{N\times N} & 0 & \mathbf{I}_{N\times N} \end{bmatrix}$$

The goal of M_1 is to target agents to drive the average limiting belief in the network as close to 1 as possible. Her optimization problem is:

$$\max_{a_i^1, i=1,\dots,n} B(a^1)$$

$$\text{s.t.} \sum_{i=1}^n a_i^1 \leq 1 \text{ and } a_i^1 \geq 0 \text{ for all } i$$

The optimal targeting strategy takes into account the following features:

1. The agent's bias.
2. The frequency with which an agent interacts with external sources.
3. The agent's position within the network.

The first two quantities are given by b_i and $\alpha_i f(a_i^1)$, respectively. Regarding the last, how does one quantify the importance of an agent in the network? In each period, each agent i receives a message from outside their peer network with probability α_i. From the influencer's perspective, it must quantify how much her message gets dispersed through the network once a given agent i receives the said message. Consider the matrix $\sum_{t=0}^\infty (D^\alpha P)^t$. The $(j,i)^{\text{th}}$ entry represents the time-discounted expected number of paths between j and i. In other words, the long-run influence i has on j. The matrix $\sum_{t=0}^\infty (D^\alpha P)^t$ can be written succinctly as $(I - D^\alpha P)^{-1}$, where I is the $N\times N$ identity matrix. Denote the vector $e^T(I - D^\alpha P)^{-1}$ as $\hat{q}$. Each component $\hat{q}_i = \sum_{j=1}^N (\sum_{t=0}^\infty (D^\alpha P)^t)_{ji}$ quantifies the total long-run influence agent i has on the rest of the network. However, with probability α_i, each agent i interacts with a source outside his peer network. Thus, the influence is scaled-down by α_i. Let q denote the "scaled-down vector". That is, $q_i = \alpha_i \hat{q}_i$. Call q **the attention-adjusted centrality vector**.[8] The average limiting belief can be decomposed into a linear sum of each of these features.

Theorem 1. *The average limiting belief in the network is* $\frac{1}{N}\sum_{i=1}^{n} q_i[(1-b_i)f(a_i^1)+b_i]$.[9] *The influencer prefers targeting those with higher attention-adjusted centrality and those with opposite bias. Formally, given optimal targeting strategy* a^{1*}:

1. *If* $(1-b_i)q_i > (1-b_j)q_j$ *then either* $a_i^{1*} > a_j^{1*}$ *OR* $a_i^{1*} = a_j^{1*} = 0$.
2. *If* $a_i^{1*} > a_j^{1*}$ *then* $(1-b_i)q_i > (1-b_j)q_j$.

The sharp characterization of the average limiting belief in the network highlights the fundamental forces at work. Unlike traditional DeGroot learning models, the average limiting belief will not be each agent's limiting belief. A consensus will not emerge because the influencer and private sources act as "stubborn nodes" in the network that never update their beliefs.

All things equal, agents with a higher attention-adjusted centrality are more valuable. The attention-adjusted centrality measure $q_i = \alpha_i \hat{q}_i$ is a weighted network centrality measure: Each agent i's contribution to the limiting belief is scaled by the amount of direct attention that the agent gives to external sources each period. Notice, though, that the influencer considers the messages agents receive from the private sources: q_i is weighted by $1-b_i$. An influencer must consider the agent's initial beliefs as those are reinforced via the residual attention paid to the private sources; the influencer must consider the agent's bias. Agents with an initial belief farther away from 1 are more important for targeting. Because the influencer faces no competition, there is less need to target agents who are already biased toward her message: such agents will receive similar messages anyways! Within the single-influencer setting, an influencer prefers targeting agents with initial beliefs *farther* from her message. In the extreme case where agents have either belief 1 or 0, all agents with a belief of 1 are ignored. That is, $a_i^1 = 0$ when $b_i = 1$.

The single-influencer setting should be interpreted as an environment where the strategic influencer faces passive competition, and so her targeting can displace the attention agents pay to their private sources. When faced with passive competition, a seller should target agents biased toward its competitor. A propagandist should target those biased in the opposite direction.

3.2 | Effect of Network Structure on Payoffs

The characterization of the influencer's payoff function in Theorem 1 leads to the natural question of which network she prefers to face. In other words, for which non-negative, row-stochastic matrices P is the influencer's optimal payoff highest? As her payoff depends on the network P through the attention-adjusted centrality vector, this question reduces to identifying the q's that maximize $\max_{a_1^1,\dots,a_N^1} \frac{1}{N}\sum_{i=1}^{N} \alpha_i q_i (1-b) f(a_i^1)$.

For simplicity of exposition, I will assume each agent has the same bias $b < 1$.[10] Heterogeneous bias will not affect the result. Now, recognize that the question above is equivalent to asking when strategic targeting is most valuable. Without knowledge of the network, the influencer will target each agent equally, securing a payoff of $(1-b)f(\frac{1}{N}) + b$. The value of knowing the network and targeting optimally is given by the difference between the payoff under optimal targeting and this benchmark: $\max_{a_1^1,\dots,a_N^1} \frac{1}{N}\sum_{i=1}^{N} \alpha_i q_i (1-b)\cdot[f(a_i^1) - f(\frac{1}{N})]$. This value is maximized precisely at q's for which $\max_{a_1^1,\dots,a_N^1} \frac{1}{N}\sum_{i=1}^{N} \alpha_i q_i (1-b) f(a_i^1)$ is maximal.

Proposition 1. *Strategic targeting is most valuable when facing star networks. It is least valuable when facing a complete network.*

The influencer prefers if the attention-adjusted centralities are concentrated among a few individuals, with the star network serving as the most extreme case. Centralities are most dispersed in complete networks, and the influencer is forced to distribute her budget equally across agents. The intuition behind the preference is that when centralities are concentrated across a small subset of agents, the influencer can expend all her resources targeting the most central agents and benefit tremendously from peer-to-peer learning.

4 | Competition

To incorporate competition, I add a second influencer, M_2, with belief 0. Influencer M_1 wishes *to maximize* the average belief, whereas M_2 wishes *to minimize* it. To provide an interpretation, consider two firms competing for customers. The initial belief represents an agent's natural, passive bias toward each firm's products. The long-run beliefs represent the long-run frequency of purchases from a given firm. In a political context, each influencer represents a rival political group. The long-run beliefs represent the long-run probability with which a given agent sides with a particular group.

When both influencers are strategic, the optimization problems for each, fixing the targeting decision of her competitor, are as follows:

M_1	M_2
$\max_{a_1^1,\dots,a_N^1} B(a^1, a^2)$ s.t. $\sum_{i=1}^{N} a_i^1 \le 1,\ \ a_i^1 \ge 0$ for each i	$\max_{a_1^2,\dots,a_N^2} 1 - B(a^1, a^2)$ s.t. $\sum_{i=1}^{N} a_i^2 \le 1,\ \ a_i^2 \ge 0$ for each i

Definition 1. A **pure strategy equilibrium** is a profile of pure strategies (a^1, a^2) such that each influencer is best-responding to her competitor's targeting strategy.

The influencers engage in a simultaneous move game where each selects a targeting strategy.

Pure strategy equilibria are guaranteed to exist. Mixed-strategy equilibria do not.[11] A trivial generalization of the proof of Theorem 1 yields the following expression for the average limiting belief in the network under any targeting profile:

$$B(a^1,a^2) = \underbrace{\frac{1}{N}\sum_{i=1}^{N} q_i f(a_i^1,a_i^2)(1-b_i)}_{\substack{\text{Gain from direct interaction}\\ \text{“Single-Influencer” Component}}} - \underbrace{\frac{1}{N}\sum_{i=1}^{N} q_i f(a_i^2,a_i^1) b_i}_{\substack{\text{Gain From Reducing}\\ \text{Competitor's Influence}}} + \underbrace{\frac{1}{N}\sum_{i=1}^{N} q_i b_i}_{\substack{\text{Avg. Belief}\\ \text{w/ no Influencers}}}$$

Looking at the expression for the average limiting belief highlights important incentives in the competition game. Both influencers weigh centrality in the same manner: All else equal, the more central an agent, the higher the marginal gain from targeting that agent. How influencers treat agents based on their initial, persistent beliefs is not immediate. An influencer's spending on agent i has two effects: it *increases* direct interaction with the agent and *decreases* her competitor's direct interaction with the agent. The former is scaled by $1 - b_i$, whereas the latter is scaled by b_i. Hence, there are benefits from targeting those with beliefs far from 1 and those with beliefs close to 1.

The influencer objective functions are reminiscent of Colonel Blotto games. In traditional Blotto games, "winning" is discrete. One can interpret this game as a Blotto game where winning is continuous, battlefields are of size q_i, and each influencer has advantages on some battlefields over others.

4.1 | Equilibrium: Unbiased Agents

Before examining competition in a setting with biased agents, it is useful to first consider the case where agents are unbiased: $b_i = \frac{1}{2}$ for all i. This setting, which can represent an undifferentiated duopoly or a political contest where agents hold no initial bias toward either party, allows us to isolate the effect of competition. I can then determine whether competition alone distorts incentives relative to the single-influencer setting, irrespective of bias.

Theorem 2. *Suppose $b_i = \frac{1}{2}$ for all i. Then, there are only pure-strategy symmetric equilibria. Moreover, if f satisfies $\frac{\partial f(a,c)}{\partial x \partial y} \le \frac{\partial f(c,a)}{\partial x \partial y}$ for $a < c$, then in any equilibrium, influencers spend more targeting central agents in the network than non-central agents.*

When agents are unbiased, the game is symmetric zero-sum. Competition leads to targeting agents symmetrically. Theorem 2 also reveals that for a large class of competition functions, all equilibria involve influencers targeting agents with high attention-adjusted centrality. This aligns with the findings in the single-influencer setting. When agents are unbiased, an influencer prefers to target more central agents: competition does not distort their incentives.

To provide intuition regarding the condition on f, suppose an influencer spends a on an agent, and her competitor spends $c > a$. The term $\frac{\partial f(a,c)}{\partial x \partial y}$ represents the effect of the competitor's spending on the marginal return in direct interaction. Now, $\frac{\partial f(c,a)}{\partial x \partial y}$ can be interpreted as the effect of the competitor's spending on one's marginal return of deterrence (i.e., how the competitor's spending affects $\frac{\partial f(c,a)}{\partial y}$). Thus, $\frac{\partial f(a,c)}{\partial x \partial y} \le \frac{\partial f(c,a)}{\partial x \partial y}$ for $a < c$ reflects the idea that overspending disincentivizes one's opponent from spending on that agent. To see this more clearly, consider M_1's objective, which is to maximize:

$$\frac{1}{N} \sum_{i=1}^{N} q_i \cdot \frac{1}{2} \underbrace{[f(a_i^1, a_i^2) - f(a_i^2, a_i^1)]}_{\text{Gain from targeting agent } i}$$

The function $h(x, y) = f(x, y) - f(y, x)$ is the "normalized gain" from targeting agent i ("normalized" because it does not include the scaling via the attention-adjusted centrality). The partial derivative of h with respect to x is the sum of the marginal return in direct interaction and the positive externality created by reducing the competitor's direct interaction. The condition on f implies $\frac{\partial h(a,c)}{\partial x \partial y} \le 0$ for $c \ge a$. In other words, influencer spending decisions are partially strategic substitutes. Consequently, $\frac{\partial h(a,a)}{\partial x} > \frac{\partial h(c,c)}{\partial x}$: when influencers are spending small amounts on an agent, there is a greater gain to increased spending. Many competition functions satisfy this property, including the classical Tullock competition function, $f(x, y) = \frac{x}{x+y+\delta}$.[12]

4.2 | Biased Agents

In the previous section, I showed that competition alone does not alter the qualitative structure of who is targeted relative to the single-influencer case. This raises the question of whether the dynamics change when agents are biased. In other words, does the combination of competition with agent bias shift targeting strategies? Do the insights from the single-influencer setting carry over, and will influencers focus on those with dissimilar beliefs?

When agents have varying biases, the game is no longer symmetric, and asymmetric equilibria emerge. To determine how influencers incorporate agents' biases in the presence of competition, I consider a class of networks that I call *balanced*.

Definition 2. A network of N agents, N even, is said to be **balanced** if there exists a bijective map $G : \{1, \ldots, N\} \longrightarrow \{1, \ldots, N\}$ with $b_i = 1 - b_{G(i)}$ and $q_i = \alpha_i \hat{q}_i = \alpha_{G(i)} \hat{q}_{G(i)} = q_{G(i)}$.

In a balanced network, for each agent i with belief b_i and attention-adjusted centrality q_i, there is a unique agent j such that $b_j = 1 - b_i$ and $\alpha_j \hat{q}_j = \alpha_i \hat{q}_i$. Individual agents may be biased in one direction or another, but there is no bias *on average*. Many networks have this structure, including symmetric core-periphery networks: networks with K highly central nodes, each connected to $\frac{N-K}{K}$ nodes of low centrality. However, a balanced network does not require symmetry of shape, merely symmetry of the attention-adjusted centrality $q_i = \alpha_i \hat{q}_i$, which is a weaker condition. Considering such networks allows me to isolate the effect of the characteristics of competition on the incentive to target like-minded agents in equilibrium.

Recall the interpretation of the competing influencers as a model of duopoly competition between two firms fighting for customers. A balanced network is an environment with two groups of customers, those leaning toward one of the firms and the other leaning toward the second firm. Within each group, agents have differing intensities of bias. The constraint on centralities ensures no one set of customers has a dominant influence over the other.

In the example below, I describe a game over a two-agent balanced network. The competition function is the classical Tullock competition function.

Example 1. Let $f(x,y) = \frac{x}{x+y+\delta}$, with $\delta \in (0,1)$. Suppose the network has two agents with initial beliefs $b_1 = 1$ and $b_2 = 0$. Assume the attention-adjusted centrality measures satisfy $q_1 = q_2 = q$. Using the Karush–Kuhn–Tucker (KKT) conditions of optimality, the following system of equations must be satisfied:

$$\frac{a_1^2}{a_1^1 + a_1^2 + \delta} = \frac{a_2^2 + \delta}{a_2^1 + a_2^2 + \delta} \text{ and } \frac{a_2^1}{a_2^1 + a_2^2 + \delta} = \frac{a_1^1 + \delta}{a_1^1 + a_1^2 + \delta}$$

$$a_1^1 + a_2^1 = a_1^2 + a_2^2 = 1$$

Solving yields the unique equilibrium: $a^1 = (\frac{1-\delta}{2}, \frac{1+\delta}{2})$ and $a^2 = (\frac{1+\delta}{2}, \frac{1-\delta}{2})$.

Consistent with the single-influencer setting, the influencers spend more of their budget targeting the agent with a differing initial belief. The particular competition function used in Example 1 incentivizes targeting agents with different beliefs, aligning with the findings in the single-influencer setting. However, this is not guaranteed. The characteristics of the equilibria are sensitive to the properties of the competition function f. The Tullock competition function incentivizes influencers to "reach across the aisle" because it does not incentivize deterrence. To formalize this, I introduce the following definition:

Definition 3. Competition is said to be **intense** if the following holds:

1. $\frac{\partial f(a,c)}{\partial x} - \frac{\partial f(c,a)}{\partial y} \geq \frac{\partial f(c,a)}{\partial x} - \frac{\partial f(a,c)}{\partial y}$ whenever $a < c$.
2. $-\frac{\partial f(c,a)}{\partial y} > \frac{\partial f(c,a)}{\partial x}$ whenever $a < c$.

To provide the intuition behind the definition, suppose an influencer is underspending on one agent and overspending on another relative to her competitor. The first condition represents a competitive incentive: There are weakly larger gains to be had from spending on agents one is underspending on than from continuing to spend on those one is overspending on. This condition is satisfied by numerous classical competition functions, such as the Tullock competition function from Example 1. The key property is the second, which the Tullock competition function does not satisfy.[13] The second condition represents the *deterrence incentive*: Spending more on the agent she is underspending on will hurt her competitor more than spending on the one she is overspending on will help herself.

Theorem 3. *Given a balanced network, if competition is intense, influencers spend more targeting agents biased toward them.*[14]

The proof of the theorem shows that for any pair of agents i and $G(i)$, each influencer spends more targeting the agent who is already biased toward her message. When competition is intense, the gain from protecting conforming agents outweighs the loss from reducing spending on agents with dissimilar beliefs. Each influencer benefits more from targeting agents that are more valuable to her competitor. Such agents are precisely the ones that are biased toward the influencer. The most powerful incentive is deterring the opposition and *protecting* one's conforming agents from being altered. Such a finding informs some of the applications highlighted in the introduction. The reason why political propagandists direct resources to target their base and firms spend money targeting customers already biased toward purchasing their product may be due to deterrence incentives.

Under what competition functions is competition intense? The properties listed in Definition 3 can be satisfied using an extension of the Tullock competition function that incorporates the notion that agents are already aware of each influencer (i.e., $f(0,0) > 0$). One example of such a function is $f(x,y) = \frac{1+x}{2+1.5x+1.5y}$.[15] This function f has the curious feature that $f(x,y) + f(y,x)$, the *total* fraction of attention accorded to influencers, is decreasing in x and y. In the context of firm marketing competitions, this phenomenon is referred to as "advertising wearout": Consumers become fatigued or desensitized to repeated advertisements. In political campaigns, it is known as "message fatigue": Voters disengage after being exposed to excessive political ads.[16] In both cases, although spending might reduce a competitor's influence, it does not fully translate into increased attention toward the advertiser. Part of the freed-up attention is redirected to the private source reinforcing the agent's bias. However, this can be a positive if the agent is biased in a favorable direction.

The conditions needed in the definition of "intense competition" are to ensure that Theorem 3 holds *independent* of the magnitude of the biases and distribution of centralities q in the network. If one had more network information, such conditions could be relaxed. For example, if network centralities are not too dispersed so that each influencer targets each agent with a fraction $\epsilon > 0$ of her budget, then $-\frac{\partial f(c,a)}{\partial y} > \frac{\partial f(c,a)}{\partial x}$ need only hold for $\epsilon \leq a < c$.

Intense competition is sufficient but not necessary for influencers to spend more, targeting like-minded agents. However, the second condition specified in the definition of intense competition is critical.

Proposition 2. *If in every balanced network, influencers spend more targeting those biased toward them, then* $-\frac{\partial f(c,a)}{\partial y} > \frac{\partial f(c,a)}{\partial x}$ *whenever* $a < c$.

Proposition 2 demonstrates that the deterrence incentive must be strong for equilibrium targeting to favor like-minded agents, *independent* of the magnitude of the biases and distribution of centralities q in the network. Importantly, if $b_i \in \{0,1\}$ for each i, then this deterrence property is both necessary and sufficient for Theorem 3 to hold. The crucial feature that leads to targeting like-minded agents is whether targeted spending can reduce the competitor's ability to influence an agent. It is not so much whether spending leads to more direct interaction with a given agent but whether it can reduce direct interaction with one's competitor.

4.3 | Effect of Network Structure on Payoffs

The interaction between bias and competition also affects how influencers assess which network structures are most advantageous. When agents are unbiased and the conditions of Theorem 2 are met, the average limiting belief in equilibrium is always $\frac{1}{2}$, independent of the network structure. However, when agents share a common bias b, each influencer prefers different network structures based on whether the bias is in their favor.

Proposition 3. *Suppose each agent has bias b. If $f(\frac{1}{N}, \frac{1}{N}) < f(1,1)$, then influencer M_1 prefers a complete (star) network over a star (complete) network whenever $b > (<)\frac{1}{2}$.*[17]

Proposition 3 stands in stark contrast to Proposition 1. When an influencer's competitor is passive, the influencer prefers to face a star network. That is not necessarily the case when both influencers are strategic. Consider the case when $b > \frac{1}{2}$: All agents are biased toward influencer M_1. If centralities are dispersed, M_1's competitor must spread out its budget, allowing M_1 to take full advantage of the agents' favorable bias. Likewise, when $b < \frac{1}{2}$, M_1 prefers if the centralities are concentrated as it can focus all of its resources on a few agents, giving it the best chance of "converting" the network.

The interpretation of the assumption that all agents share a common bias is that it reflects a setting where one of the influencers is an incumbent that has acted as a single-influencer for a length of time, resulting in a homogeneous bias toward it. The implication of Proposition 3 is that an incumbent is in a stronger competitive position against future entrants when the agents are part of a complete network, and no single agent has a dominant influence over his peers.

5 | Final Remarks

Many environments involve influencers attempting to shape beliefs. Individuals, though, often form beliefs not only from external targeting but also from their peers. One of the article's goals was to develop a tractable model to understand these settings better. Agents in my model learn via a DeGroot heuristic, but what distinguishes my model from other DeGroot learning models are the non-strategic *and* strategic external sources. Including these external sources allows for the persistence of agents' bias, ensuring agents do not reach a consensus belief in the long run. In traditional DeGroot models of belief formation, all agents share the same belief in the long-run, and so influencers are solely concerned with an agent's location in the network (Golub and Jackson 2010; Golub and Sadler 2016).

An implicit assumption built into the DeGroot updating rule is that all agents in the network are informed about the relevant issue. Therefore, although the applications of my model are to firm marketing competitions and propaganda campaigns, it is important to note that it is best suited for settings where consumers are already familiar with the brands or where voters are aware of the political event and understand the general left- or right-wing perspectives. A common criticism of the DeGroot learning rule is its simplicity and how agents do not account for the repetition of information. However, accounting for such repetition requires significant computing power, and arguments based on bounded rationality support the use of such heuristics (DeMarzo et al. 2003). In fact, Chandrasekhar et al. (2020) provide empirical evidence demonstrating that it mirrors observed patterns of information-sharing behavior in communication networks. Additionally, Grimm and Mengel (2020) demonstrate via experiments that DeGroot learning captures how agents form opinions about the value of an unknown state.

The belief updating rule in this model is mathematically equivalent to other processes. For instance, suppose learning is stochastic: P^*_{ij} denotes the probability of interaction between i and j, and each time communication takes place, agent i adopts the belief held by the party she communicates with. Then, $\lim_{t\to\infty} {P^*}^t b$ represents the expected belief held by each agent in the long run. Another communication and updating protocol yielding the same structure involves agents and external sources sharing binary articles (0 or 1), with beliefs in period t equal to the fraction of articles received with a value of 1. If b_i is the initial frequency at which agent i shares an article with a value of 1, and sharing frequencies in period $t+1$ are equal to b^t, then $\lim_{t\to\infty} {P^*}^t b$ is the long-run fraction of articles with a value of 1 that agents have received.

Given the model and its applications, choosing to use long-run limiting beliefs as the influencer's objective is natural. The model, though, does provide a framework to explore targeting behavior when influencers have other objectives, which may be interesting for future research. For example, if influencer M_j is focused on long-run awareness, it would want to maximize the quantity $\lim_{t\to\infty} \frac{1}{N} \cdot \sum_{i=1}^{N} {P^*}^t_{i,N+j}$, which measures how much agents incorporate the influencer into their long-run beliefs. Similarly, in other applications, it might be that the influencer only cares about agent beliefs in so far as those beliefs cross a specific threshold. Crucially, the model in this article is sufficiently tractable to analyze *competitive* settings between influencers with such objectives.

5.1 | Related Works

My article fits into the theoretical literature on opinion dynamics, offering a model of learning that incorporates both DeGroot learning and the persistence of agents' initial beliefs.[18] The learning process in my model is related to that described in Friedkin and Johnsen (1999), where agents are embedded in a network, learn from their peers, and are heterogeneous in their susceptibility to interpersonal influence. A lack of consensus in the limit arises when some agents are not susceptible to peer influence.[19] My model can be viewed similarly by interpreting $1-\alpha_i$ as agent i's susceptibility to his peers and α_i as his susceptibility to the external sources. However, the influencers in my model can choose whom to link to and the intensity of the link, thereby making agent susceptibility toward external sources endogenous.

The idea of influencer targeting to spread information in a network is related to the research on "seeding" a network (e.g., Kempe et al. 2003, 2005; Banerjee et al. 2013, 2019; Kim et al. 2015). However, my article incorporates *strategic competition in diffusion*. The addition of competition in seeding causes influencers to consider how seeding reduces the influence of their rivals, a force absent in a single-influencer seeding setting.

Mostagir et al. (2022) also examine how a single influencer manipulates long-run beliefs of agents in a network when agents receive impressions across multiple external sources. A crucial distinction is that the influencer is not budget constrained and only benefits from targeting if an agent's belief reaches a particular threshold. One can view Mostagir et al. (2022) through the lens of my model by changing the influencer's objective function from the average limiting belief to a threshold objective and replacing the budget constraint with a marginal cost associated with spending. Then, when there is no competition, an influencer may find it optimal not to spend resources targeting (e.g., the targeting necessary to push beliefs above the threshold is too costly). However, my model highlights the importance of competition even with the threshold objective. In the absence of competition, an influencer may refrain from targeting; however, the presence of competition can encourage her to spend strategically to reduce *the rival's ability to manipulate beliefs*.

The most closely related works are Bimpikis et al. (2016) and Goyal et al. (2019), which study competitive diffusion between two firms on a network. However, agent bias is not persistent, and firms only care about the average fraction of "impressions" generated. Specifically, Bimpikis et al. (2016) is a special case of my framework when the influencer's objective is to maximize the long-run weights agents place on her (i.e., M_1 maximizing the average of the elements in the $(N+1)^{th}$ column of $\lim_{t\to\infty} P^{*t}$). An influencer with this objective is agnostic about how agents interact with other external sources. In my model, influencers must be concerned with the *distribution* of impressions generated and the *distribution* of long-run weights across *all external sources*. This distinction is significant not just at a technical level but in terms of applications. For example, suppose agents make binary choices based on their beliefs about a state. An agent's limiting belief is based on the entire distribution of messages he receives, not just the fraction of messages received directly and indirectly from an influencer.

When influencers target like-minded agents in my model, it is due to an incentive to deter one's competitor. This particular force contrasts my result with Sadler (2023), which examines opinion dynamics in a single-influencer setting. Sadler (2023) identifies conditions under which the influencer targets her base, but this is due to risk-aversion on the part of the influencer. In my model, influencers are risk-neutral, but appeals to the base occur due to competition.

Acknowledgments

I thank Joyee Deb and three anonymous referees for their valuable comments. For helpful discussions and suggestions, I thank Kostas Bimpikis, Matthew Elliott, Matthew Gentzkow, Ben Golub, Sanjeev Goyal, Matthew Jackson, Amin Saberi, Eduard Talamás, and José Tudón. I would also like to thank participants of the 2021 International Industrial Organization Conference and 2022 Annual Conference on Network Science and Economics. This work was supported in part by the ERC Grant #757229 under the European Union's Horizon 2020 research and innovation program.

Endnotes

[1] See Statista: Third Party Audience Data and Statista: Digital Advertising Spending.

[2] https://www.emarketer.com/press-releases/2024-political-ad-spending-will-jump-nearly-30-vs-2020/

[3] An alternative interpretation of private sources is that they are simply a modeling device to ensure the agent always places some weight on their initial belief.

[4] Mixed strategies would correspond to a probability measure over $\{a^1 \in [0,1]N \mid \sum a_i^1 \leq 1\}$. In the single-influencer setting, the strategy space can be restricted to pure strategies because f is concave.

[5] The inclusion of the competition function contrasts my model with Grabisch et al. (2018). In that article, influencer strategies are restricted to the formation of a single link in the network, and the effect of this link formation is fixed. I allow influencers to choose both the breadth and intensity of their targeting.

[6] If $\alpha_i = 0$, meaning agents do not interact with any external sources, then beliefs at time t are $P^t b$ as in the classic DeGroot learning models.

[7] I show the quantity $B(a^1, a^2)$ is well-defined in the proof of Theorem 1.

[8] Related is Katz–Bonacich centrality (Bonacich 1987; Bloch et al. 2023) and the DeGroot centrality measure in Mostagir et al. (2022). In the latter, influencer targeting is binary (to target or not to target) and the frequency of interaction conditional on targeting is fixed. As a result, the amount of attention agents devote to external sources is endogenous. In mine, the available attention toward external sources is fixed, but the distribution of attention is not. Hence, my centrality measure is static: it takes into account transmission within the peer network scaled by the amount of available attention directed outside the peer network.

[9] If the set of private sources agent i interacts with directly has average belief ρ_i, then the average limiting belief in the network is $\frac{1}{N}\sum_{i=1}^n q_i[(1-\rho_i)f(a_i^1)+\rho_i]$.

[10] If $b = 1$, the average limiting belief will be 1, independent of the influencer's targeting decisions.

[11] See Lemma 2.

[12] This competition function has been employed in a number of areas, including the economics of advertising, tournaments, and political economy—see Corchón (2007) for a survey.

[13] To see this, notice that $\frac{\partial f(c,a)}{\partial x} = \frac{a+\delta}{(c+a+\delta)^2}$ and $-\frac{\partial f(c,a)}{\partial y} = \frac{c}{(c+a+\delta)^2}$. For $a \in (c-\delta, c)$, $\frac{\partial f(c,a)}{\partial x} > -\frac{\partial f(c,a)}{\partial y}$.

[14] Analyzing the relationship between targeting and bias requires controlling for centrality. If not, there is a risk that network centrality dominates equilibrium behavior. Specifically, if the centrality of a subset of agents significantly exceeds that of other agents, then no influencer would spend more targeting agents outside that subset than within it. In the extreme case, where $\max q_i \approx N$, influencers would allocate nearly their entire budget to a single agent. The balanced network assumption serves as an appropriate benchmark because it avoids this effect.

[15] Competition is intense for any $f(x,y) = \frac{a+x}{b+c\cdot x+c\cdot y}$, where a, b, and c are positive constants with $ac \in (\frac{b}{2}, b)$.

[16] Blair (2000) and Lu (2022).

[17] The condition on function f is not stringent. For example, any function of the form $f(x,y) = \frac{c(x)}{c(x)+c(y)+\delta}$ where $c(\cdot)$ increasing and $c(0) = 0$, satisfies the property.

[18] Molavi et al. (2018) and Dasaratha et al. (2023) provide microfoundations for the DeGroot learning rule.

[19] Such agents are equivalent to the "stubborn" agents in Acemoglu et al. (2010) and Yildiz et al. (2013).

[20] As $\hat{P} + \psi e^T$ is row-stochastic and v is a unit vector, $\sum_{j=1}^N v_j \sum_{i=1}^N (\hat{P}_{ji} + \Psi_j) = \sum_{j=1}^N v_j$.

[21] Let $l^* := \max\{i : a_i > 0\}$. The existence of k^* guarantees $q_1' > q_1$. Therefore, the claim is trivially true if $l^* = 1$, as that would mean $a_1 = 1$ and $a_i = 0$ for all $i \geq 2$. If $l^* > 1$, then the KKT conditions in (A.1) imply $\frac{q_i'}{q_j'} = \frac{q_i}{q_j}$ for all $i, j \leq l^*$. As $q_1' > q_1$, this means $q_i' > q_i$ for all $i \in \{1, \ldots, l^*\}$.

[22] By Lemma 4, $i^* \geq 2$, and so such a δ exists.

[23] The influencer's optimal payoff is maximal when $\max_i q_i = N$. Although the star network satisfies this property, other network structures can also satisfy it. For instance, consider a tree network where the root node puts full weight on itself, and all other nodes place full weight on their parent node.

References

Acemoglu, D., G. Como, F. Fagnani, and A. Ozdaglar. 2010. "Opinion Fluctuations and Disagreement in Social Networks." *Mathematics of Operation Research* 38, no. 1: 1–27.

Banerjee, A., A. Chandrasekhar, E. Duflo, and M. O. Jackson. 2013. "The Diffusion of Microfinance." *Science* 341, no. 6144: 1236498.

Banerjee, A., A. Chandrasekhar, E. Duflo, and M. O. Jackson. 2019. "Gossip: Identifying Central Individuals in a Social Network." *Review of Economic Studies* 86, no. 6: 2453–2490.

Beaman, L., A. BenYishay, J. Magruder, and A. M. Mobarak. 2021. "Can Network Theory-Based Targeting Increase Technology Adoption?" *American Economics Review* 111, no. 6: 1918–1943.

Bergemann, D., and A. Bonatti. 2011. "Targeting in Advertising Markets: Implications for Offline Versus Online Media." *The RAND Journal of Economics* 42, no. 3: 417–443.

Bimpikis, K., A. Ozdaglar, and E. Yildiz. 2016. "Competitive Targeted Advertising Over Networks." *Operations Research* 64, no. 3: 705–720.

Blair, M. H. 2000. "An Empirical Investigation of Advertising Wearin and Wearout." *The Journal of Advertising Research* 40, no. 6: 95–100.

Bloch, F., M. Jackson, and P. Tebaldi. 2023. "Centrality Measures in Networks." *Social Choice and Welfare* 61: 413–453.

Bonacich, P. 1987. "Power and Centrality: A Family of Measures." *American Journal of Sociology* 92: 1170–1182.

Chandrasekhar, A., H. Larreguy, and J. Xandri. 2020. "Testing Models of Social Learning on Networks: Evidence from Two Experiments." *Econometrica* 88: 1–32.

Coleman, J. S., E. Katz, and H. Menzel. 1966. Medical Innovation: A Diffusion Study. Bobbs-Merrill Co.

Corchón, L. 2007. "The Theory of Contests: A Survey." *Review of Economic Design* 11: 69–100.

Dasaratha, K., N. Hak, and B. Golub. 2023. "Learning from Neighbors about a Changing State." *Review of Economic Studies* 90, no. 5: 2326–2369.

DeMarzo, P., D. Vayanos, and J. Zwiebel. 2003. "Persuasion Bias, Social Influence, and Uni-Dimensional Opinions." *Quarterly Journal of Economics* 118: 909–968.

Fainmesser, I., and A. Galeotti. 2015. "Pricing Network Effects." *The Review of Economic Studies* 83, no. 1: 165–198.

Fainmesser, I., and A. Galeotti. 2020. "Pricing Network Effects: Competition." *American Economic Journal: Microeconomics* 12, no. 3: 1–32.

Friedkin, N., and E. Johnsen. 1999. "Social Influence Networks and Opinion Change." *Advances in Group Processes* 16: 1–29.

Galeotti, A., and S. Goyal. 2009. "Influencing the Outlets: A Theory of Strategic Diffusion." *The RAND Journal of Economics* 40, no. 3: 509–532.

Golub, B., and M. O. Jackson. 2010. "Naive Learning in Social Networks and the Wisdom of Crowds." *American Economic Journal: Microeconomics* 2, no. 1: 112–149.

Golub, B., and E. Sadler. 2016. "Learning in Social Networks." In *The Oxford Handbook of the Economics of Networks*, 504–542. Oxford Academic.

Goyal, S., H. Heidari, and M. Kearns. 2019. "Competitive Contagion in Networks." *Games and Economic Behavior* 113: 58–79.

Grabisch, M., A. Mandel, A. Rusinowska, and E. Tanimura. 2018. "Strategic Influence in Social Networks." *Mathematics of Operations Research* 43, no. 1: 29–50.

Huckfeldt, R., P. E. Johnson, and J. Sprague. 2004. *Political Disagreement: The Survival of Diverse Opinions within Communication Networks*. Cambridge University Press.

Iyengar, R., C. Van den Bulte, and T. W. Valente. 2011. "Opinion Leadership and Social Contagion in New Product Diffusion." *Marketing Science* 30, no. 2: 195–212.

Iyer, G., D. Soberman, and J. Miguel Villas-Boas. 2005. "The Targeting of Advertising." *Marketing Science* 24, no. 3: 461–476.

Kempe, D., J. M. Kleinberg, and E. Tardos. 2003. "Maximizing the Spread of Influence through a Social Network." In *KDD*, 137–146. Association for Computing Machinery.

Kempe, D., J. M. Kleinberg, and E. Tardos. 2005. "Influential Nodes in a Diffusion Model for Social Networks." In *ICALP*, 1127–1138. Springer.

Kim, D. A., A. R. Hwong, D. Stafford, et al. 2015. "Social Network Targeting to Maximize Population Behavior Change: A Cluster Randomized Controlled Trial." *The Lancet* 386, no. 9989: 145–153.

Lever, C. 2010. "Strategic Spending in Voting Competitions with Social Networks." Working paper 2010-16, Banco de México.

Lu, H. 2022. "The Role of Repeated Exposure and Message Fatigue in Influencing Willingness to Help Polar Bears and Support Climate Change Mitigation." *Science Communication* 44, no. 4: 475–493.

Mostagir, M., A. Ozdaglar, and J. Siderius. 2022. "When is Society Susceptible to Manipulation?" *Management Science* 68, no. 10: 7153–7175.

Molavi, P., A. Tahbaz-Salehi, and A. Jadbabaie. 2018. "A Theory of Non-Bayesian Social Learning." *Econometrica* 86: 445–490.

Sadler, E. 2023. "Influence Campaigns." *American Economic Journal: Microeconomics* 15, no. 3: 271–304.

Trusov, M., R. E. Bucklin, and K. Pauwels. 2009. "Effects of Word-of-Mouth versus Traditional Marketing: Findings from an Internet Social Networking Site." *Journal of Marketing* 73, no. 5: 90–102.

Yildiz, E., A. Ozdaglar, D. Acemoglu, A. Saberi, and A. Scaglione. 2013. "Binary Opinion Dynamics with Stubborn Agents." *ACM Transactions on Economics and Computation* 1, no. 4: 1–30.

Appendix A: Main Proofs

A.1 | Proofs: Targeting and Equilibrium

To compute $B^*(a^1)$, I need to first compute $\lim_{t\to\infty} {P^*}^t$. Simple matrix algebra yields:

$$
{P^*}^t = \begin{bmatrix} (D^\alpha P)^t & \left(\sum_{i=0}^t (D^\alpha P)^i\right)\cdot \alpha f(a^1) & \left(\sum_{i=0}^t (D^\alpha P)^i\right)\cdot D^S \\ \mathbf{0}_{1\times N} & 1 & \mathbf{0}_{1\times N} \\ \mathbf{0}_{N\times N} & \mathbf{0}_{N\times 1} & \mathbf{I}_{N\times N} \end{bmatrix}
$$

Now, $\lim_{t\to\infty}\sum_{i=0}^t (D^\alpha P)^i = (I - D^\alpha P)^{-1}$, so the only term left to calculate is $\lim_{t\to\infty}(D^\alpha P)^t$.

Lemma 1. *If $\hat{P} = D^\alpha P$ for some $\alpha \neq \mathbf{0}$, then* $\lim_{t\to\infty}\hat{P}^t = 0$.

Proof. $\hat{P}$ is substochastic with at least one row sum strictly less than 1. As P is aperiodic and strongly connected, $\hat{P}$ is *irreducible* and there exists $n \in \mathbb{N}$ such that $\hat{P}^n$ has positive entries. By the Perron–Frobenius theorem, there exists $\lambda > 0$ such that λ is the largest eigenvalue of $\hat{P}$ and the associated unit *left* eigenvector v of $\hat{P}$ is strictly positive.

Let $\Psi \in \mathbb{R}^N_+$ be the positive vector such that $\Psi_j = \frac{1}{N}(1 - \sum_{i=1}^N \hat{P}_{ji})$. Thus, $\hat{P} + \Psi e^T$ is row-stochastic, where e is a column vector of 1's. Now:

$$
\lambda v = v^T\hat{P} \implies \lambda v_i = \sum_{j=1}^N \hat{P}_{ji} v_j \text{ for each } i \implies \lambda = \sum_{i=1}^N\sum_{j=1}^N \hat{P}_{ji}v_j
$$

$$
= \sum_{i=1}^N\sum_{j=1}^N(\hat{P}_{ji} - \Psi_j + \Psi_j)v_j = \sum_{i=1}^N\sum_{j=1}^N(\hat{P}_{ji} + \Psi_j)v_j - \sum_{i=1}^N\sum_{j=1}^N \Psi_j v_j = \sum_{i=1}^N\sum_{j=1}^N(\hat{P}_{ji} + \Psi_j)v_j - N(\Psi\cdot v)
$$

$= \sum_{j=1}^N v_j \sum_{i=1}^N(\hat{P}_{ji} + \Psi_j) - N(\Psi\cdot v) = 1 - N(\Psi \cdot V) < 1$, because Ψ is non-zero and non-negative.[20] Recognize that $v^T\hat{P} = \lambda v \implies v^T\hat{P}^t = \lambda^t v \implies \lim_{t\to\infty} v\hat{P}^t = 0$. As v is positive and $\hat{P}$ is non-negative, $\lim_{t\to\infty}\hat{P}^t = 0$. □

Proof of Theorem 1(i): Applying Lemma 1:

$$
\lim_{t\to\infty} {P^*}^t = \begin{bmatrix} \mathbf{0}_{N\times N} & (I - D^\alpha P)^{-1}\cdot \alpha f(a^1) & (I - D^\alpha P)^{-1}\cdot D^S \\ \mathbf{0}_{1\times N} & 1 & \mathbf{0}_{1\times N} \\ \mathbf{0}_{N\times N} & \mathbf{0}_{N\times 1} & \mathbf{I}_{N\times N} \end{bmatrix}
$$

$$
\implies B(a^1) = \frac{1}{N}e^T\left[\lim_{t\to\infty} P^{*t} b\right]_{N\times 1} = \frac{1}{N}e^T(I - D^\alpha P)^{-1}\cdot\left[\alpha f(a^1) + \alpha(1 - f(a^1))b\right]
$$

$$
= \frac{1}{N}\hat{q}\cdot\left[\alpha f(a^1) + \alpha(1 - f(a^1))b\right] = \frac{1}{N}\sum_{i=1}^N \alpha_i\hat{q}_i[(1-b_i)f(a_i^1) + b_i] = \frac{1}{N}\sum_{i=1}^N q_i[(1-b_i)f(a_i^1) + b_i]
$$

Proof of Theorem 1(ii): Given the closed-form expression for $B(a^1)$, the KKT conditions for optimality provide the following characterization of the optimal targeting strategy:

$$
a_i^{1*} = \begin{cases} a_i^1 & \text{s.t. } \frac{1}{N}(1-b_i)q_i\frac{\partial f(a_i^1)}{\partial x} = \mu \\ 0 & \text{if } \frac{1}{N}(1-b_i)q_i\frac{\partial f(0)}{\partial x} = \mu - \lambda_i, \quad \lambda_i \geq 0 \end{cases} \tag{A.1}
$$

Using (A.1), concavity of f implies that if $(1-b_i)q_i > (1-b_j)q_j$, then $a_i^{1*} > a_j^1$ whenever $a_j^1 > 0$.

Similarly, if $a_i^{1*} > a_j^{1*}$, then it follows from (A.1) that $\frac{1}{N}(1-b_i)q_i\frac{\partial f(a_i^{1*})}{\partial x} \geq \frac{1}{N}(1-b_j)q_j\frac{\partial f(a_j^{1*})}{\partial x}$. As, $\frac{\partial f(a_i^{1*})}{\partial x} < \frac{\partial f(a_j^{1*})}{\partial x}$ by concavity of f, it necessarily means $(1-b_i)q_i > (1-b_j)q_j$. □

Lemma 2. *There is no mixed-strategy equilibrium.*

Proof. Define $h_i(x,y) = (1-b_i)f(x,y) - b_i f(y,x)$ for any $x, y \in [0,1]$. Given pure strategies $a^1, a^2 \in \left\{z : z \in \mathbb{R}^N, z_i \geq 0, \sum_{i=1}^N z_i = 1\right\}$, the payoff to M_1 is $B(a^1, a^2) = \frac{1}{N}\sum_{i=1}^N q_i[h_i(a_i^1, a_i^2) + b_i]$, whereas the payoff to M_2 is $1 - B(a^1, a^2)$. The game is obviously zero-sum.

Suppose there is a mixed-strategy equilibrium and M_2 uses mixed strategy σ_2 over the simplex. It must be that M_1 is indifferent between all actions in the support of her strategy and prefers the actions in the support to those outside of it. Suppose M_1 plays a pure strategy a^1 where $a_i^1 = \mathbb{E}_{\sigma_2}[a_i^2]$. By Jensen's inequality, M_1's payoff is:

$$
\frac{1}{N}\int\sum_{i=1}^N q_i h_i(a_i^1, a_i^2)d\sigma_2 = \frac{1}{N}\sum_i q_i\int h_i(a_i^1, a_i^2)d\sigma_2(a_i^2) > \frac{1}{N}\sum_i q_i h_i(a_i^1, a_i^1) = \frac{1}{N}\sum_{i=1}^N q_i b_i
$$

Thus, any mixed strategy equilibrium must guarantee M_1 a payoff strictly greater than $\frac{1}{N}\sum_{i=1}^N q_i b_i$. By symmetry, M_2 must be guaranteed a payoff strictly greater than $1 - \frac{1}{N}\sum_{i=1}^N q_i b_i$. However, the sum of their payoffs would then be strictly greater than 1, which is impossible. Thus, no mixed-strategy equilibrium exists. □

Proof of Theorem 2: The action set of each influencer is convex and compact, and $B(a^1, a^2)$ is continuous in each argument, concave in a^1, and convex in a^2. Therefore, a pure strategy equilibrium exists. By Lemma 2, no mixed strategy equilibrium exists. As the game is symmetric zero-sum, all equilibria are symmetric, and so I suppress dependence of the targeting strategy on the index of the influencer. Consider any equilibrium (a, a). Suppose $a_i \leq a_j$ and $q_i > q_j$. It follows that:

$$q_i\left(\frac{\partial f(a_i, a_i)}{\partial x} - \frac{\partial f(a_i, a_i)}{\partial y}\right) > q_i\left(\frac{\partial f(a_i, a_j)}{\partial x} - \frac{\partial f(a_j, a_i)}{\partial y}\right) > q_i\left(\frac{\partial f(a_j, a_j)}{\partial x} - \frac{\partial f(a_j, a_j)}{\partial y}\right)$$
$$> q_j\left(\frac{\partial f(a_j, a_j)}{\partial x} - \frac{\partial f(a_j, a_j)}{\partial y}\right)$$

This violates the KKT conditions of optimality unless $a_i = a_j = 0$. ☐

Lemma 3. *In a balanced network, if (a^1, a^2) is a pure strategy equilibrium, then $a_i^1 = a_{G(i)}^2$.*

Proof. Let $A_j = \{a \in [0,1]^N \mid \sum a_i \leq 1\}$ denote M_j's strategy set, and let $\pi_j : A_1 \times A_2 \longrightarrow [0,1]$ denote M_j's payoff function. Notice $\pi_1(x, y) = 1 - \pi_2(x, y)$, and so the game is zero-sum. Because M_j's payoff function is concave in her strategy, there is at least one pure-strategy equilibrium.

Given a balanced network, let G denote the corresponding function that maps each agent to her counterpart. One can view G as a permutation on $\{1, \dots, N\}$. In an abuse of notation, given any vector $x \in \mathbb{R}^N$, define $G(x) = (x_{G(1)}, \dots, x_{G(N)})$. Recognize that $G \circ G$ is the identity operator and $\pi_1(x, y) = \pi_2(G(y), G(x))$. Thus, if (x, y) is an equilibrium, $(G(y), G(x))$ must also be an equilibrium. Furthermore, $\pi_j(x, G(x)) = \frac{1}{2}$ for any x, which means that any pure-strategy equilibrium must yield payoffs of $\frac{1}{2}$ to each influencer.

Suppose there is an equilibrium (x, y) such that $y \neq G(x)$. This implies that $\pi_1(x, y) = \pi_1(x, G(x)) = \frac{1}{2} \implies \pi_2(x, y) = \pi_2(x, G(x)) = \frac{1}{2}$. However, π_2 is concave in its second argument $\implies \pi_2(x, \lambda y + (1-\lambda)G(x)) > \frac{1}{2}$ for some $\lambda \in (0, 1)$. This contradicts the assumption that (x, y) is an equilibrium. Thus, any equilibrium must be of the form $(x, G(x))$. ☐

Proof of Theorem 3: There are only pure strategy equilibria (Lemma 2). Consider any equilibrium (a^1, a^2). Lemma 3 implies $a^2 = G(a^1)$. Suppose there is an agent i with $b_i > \frac{1}{2}$ such that $a_i^1 < a_j^1$ for $j = G(i)$. By the KKT conditions of optimality:

$$(1-b_i)\frac{\partial f(a_i^1, a_i^2)}{\partial x} - b_i\frac{\partial f(a_i^2, a_i^1)}{\partial y} \leq (1-b_j)\frac{\partial f(a_j^1, a_j^2)}{\partial x} - b_j\frac{\partial f(a_j^2, a_j^1)}{\partial y}$$

$$\implies (1-b_i)\frac{\partial f(a_i^1, a_i^2)}{\partial x} - b_i\frac{\partial f(a_i^2, a_i^1)}{\partial y} \leq b_i\frac{\partial f(a_i^2, a_i^1)}{\partial x} - (1-b_i)\frac{\partial f(a_i^1, a_i^2)}{\partial y}$$

$$\implies b_i\left(-\frac{\partial f(a_i^2, a_i^1)}{\partial y} - \frac{\partial f(a_i^2, a_i^1)}{\partial x}\right) \leq (1-b_i)\left(-\frac{\partial f(a_i^1, a_i^2)}{\partial y} - \frac{\partial f(a_i^1, a_i^2)}{\partial x}\right)$$

Now, $-\frac{\partial f(a_i^2, a_i^1)}{\partial y} - \frac{\partial f(a_i^2, a_i^1)}{\partial x} > 0$ and $-\frac{\partial f(a_i^2, a_i^1)}{\partial y} - \frac{\partial f(a_i^2, a_i^1)}{\partial x} + \frac{\partial f(a_i^1, a_i^2)}{\partial y} + \frac{\partial f(a_i^1, a_i^2)}{\partial x} \geq 0$. As $b_i > \frac{1}{2}$, it follows that $b_i > 1 - b_i$:

$$\implies b_i\left(-\frac{\partial f(a_i^2, a_i^1)}{\partial y} - \frac{\partial f(a_i^2, a_i^1)}{\partial x}\right) > (1-b_i)\left(-\frac{\partial f(a_i^1, a_i^2)}{\partial y} - \frac{\partial f(a_i^1, a_i^2)}{\partial x}\right)$$

This is a contradiction given the conditions on f as a result of competition being intense. Thus, it must be that $a_i^1 > a_{G(i)}^1$ in equilibrium. A symmetric argument demonstrates that $a_{G(i)}^2 > a_i^2$. Each influencer spends more targeting the agent with a similar belief. ☐

Proof of Proposition 2: If the influencer targets like-minded agents in all networks, then, in particular, they must do so when the initial beliefs of agents are such that $b_i \in \{0, 1\}$ for all i. Thus, let us consider only balanced networks with such initial beliefs. Consider two agents i and j such that $j = G(i)$ and $b_i = 1$. Let total spending be $T(i, j)$ on these agents by an influencer in equilibrium (as the network is balanced, both influencers spend the same total amount on each of these agents). Now, for any $\epsilon \in [0, 1]$, if the centralities of agents i and j are sufficiently small, then $T(i, j) \leq \epsilon$. Likewise, if the centralities of agents i and j are sufficiently large, $T(i, j) \geq \epsilon$. Hence, there exists networks such that for any $Z \in [0, 1]$, $T(i, j) = Z$.

Given a network with $T(i, j) = Z$, the KKT conditions imply that in equilibrium:

$$b_i\left(-\frac{\partial f(a_i^2, a_i^1)}{\partial y} - \frac{\partial f(a_i^2, a_i^1)}{\partial x}\right) \leq 0$$

In any equilibrium, the influencers spend more on conforming agents, and so the expression in the parentheses is negative whenever $a_i^1 < a_i^2$. Because $a_i^2 = a_j^1 = Z - a_i^1$, it follows that:

$$-\frac{\partial f(a_i^2, a_i^1)}{\partial y} - \frac{\partial f(a_i^2, a_i^1)}{\partial x} < 0 \text{ for } a_i^1 < a_i^2 \iff \frac{\partial f(Z-a, a)}{\partial y} - \frac{\partial f(Z-a, a)}{\partial x} < 0 \text{ for } a < \frac{Z}{2}$$

As this must hold for all $Z \in [0, 1]$, the result follows. ☐

A.2 | Proofs: Network Structure

To prove Proposition 1, I use the following two lemmas. The first shows that the sum of the entries of an attention-adjusted centrality vector is N. The second shows that the influencer's optimal payoff is higher when the centralities are concentrated rather than dispersed. Within the proofs, I assume that for every attention-adjusted centrality vector $q = (q_1, \dots, q_N)$ that $q_1 \geq q_2 \geq \dots \geq q_N$. This is without loss.

Lemma 4. *If q is an attention-adjusted centrality vector, then $\sum_{i=1}^{N} q_i = N$.*

Proof. Recall $q = e^T(I - D^\alpha P)^{-1}(I - D^\alpha)$. Expanding $(I - D^\alpha P)^{-1}$ as a power series yields:

$$q = e^T \cdot \sum_{i=0}^{\infty} (D^\alpha)^i P^i \cdot (I - D^\alpha) = e^T \cdot \left[I + \sum_{i=1}^{\infty} (D^\alpha)^i P^{i-1} (P - I)^i \right]$$

$$\Longrightarrow \sum_{i=1}^{N} q_i = e^T \cdot \left[I + \sum_{i=1}^{\infty} (D^\alpha)^i P^{i-1} (P - I)^i \right] \cdot e = N + \sum_{i=1}^{\infty} (D^\alpha)^i P^{i-1} \underbrace{(P - I)^i e}_{=0} = N \qquad \square$$

Lemma 5. *Let q and q' be two attention-adjusted centrality vectors. If there exists k^* such that $q'_i > q_i$ for $i \in \{1, \dots, k^*\}$ and $q'_i \leq q_i$ for $i \in \{k^* + 1, \dots, N\}$, then:*

$$\max_{a_1^1, \dots, a_N^1} \frac{1}{N} \sum_{i=1}^{N} q'_i \left[(1 - b) f(a_i^1) + b \right] > \max_{a_1^1, \dots, a_N^1} \frac{1}{N} \sum_{i=1}^{N} q_i \left[(1 - b) f(a_i^1) + b \right]$$

Proof. For notational convenience, set $h(x) = (1 - b) f(x) + b$. Let a' and a denote the optimal targeting strategy when the attention-adjusted centrality vector is q' and q, respectively.

Case #1: $a' = a$

First, recognize that $q'_i > q_i$ whenever $a_i > 0$.[21] It follows that:

$$\frac{1}{N} \sum_{i=1}^{N} q'_i h(a_i) = \sum_{i=1}^{l^*} q'_i h(a_i) + \frac{1}{N} \cdot (N - \sum_{i=1}^{l^*} q'_i) h(0)$$

$$= \frac{1}{N} \sum_{i=1}^{l^*} q_i h(a_i) + \frac{1}{N} \sum_{i=1}^{l^*} (q'_i - q_i) h(a_i) + \frac{1}{N} \cdot (N - \sum_{i=1}^{l^*} q'_i) h(0)$$

$$> \frac{1}{N} \sum_{i=1}^{l^*} q_i h(a_i) + \frac{1}{N} \cdot \left[\sum_{i=1}^{l^*} (q'_i - q_i) + N - \sum_{i=1}^{l^*} q'_i \right] h(0)$$

$$= \frac{1}{N} \sum_{i=1}^{l^*} q_i h(a_i) + \frac{1}{N} (N - \sum_{i=1}^{l^*} q_i) h(0) = \frac{1}{N} \sum_{i=1}^{N} q_i h(a_i)$$

Case #2: $a' \neq a$

By optimality of a':

$$\frac{1}{N} \sum_{i=1}^{N} q'_i h(a'_i) > \frac{1}{N} \sum_{i=1}^{N} q'_i h(a_i) = \frac{1}{N} \sum_{i=1}^{N} q_i h(a_i) + \frac{1}{N} \left(\sum_{i=1}^{k^*} (q'_i - q_i) h(a_i) - \sum_{i=k^*+1}^{N} (q_i - q'_i) h(a_i) \right) \tag{A.2}$$

$$\geq \frac{1}{N} \sum_{i=1}^{N} q_i h(a_i) + \frac{1}{N} \cdot h(a_{k^*}) \cdot \left(\sum_{i=1}^{k^*} (q'_i - q_i) - \sum_{i=k^*+1}^{N} (q_i - q'_i) \right) \tag{A.3}$$

Inequality (A.3) follows from Theorem 1: $q_i \geq q_j$ for $i < j \Longrightarrow a_i \geq a_j$. Lemma 4 implies $\sum_{i=1}^{k^*} (q'_i - q_i) - \sum_{i=k^*+1}^{N} (q_i - q'_i) = 0$, and so (A.3) is greater than or equal to $\frac{1}{N} \sum_{i=1}^{N} q_i h(a_i)$. $\square$

Proof of Proposition 1: Consider an attention-adjusted centrality vector $q = (q_1, \dots, q_N)$. Lemma 4 implies $q_1 \in [1, N]$. Let $i^* = \max\{i : q_i > 0\}$ and $j^* = \max\{j : q_j = q_1\}$.

<u>Star Network:</u>

Suppose $q_1 < N$. Fix a $\delta \in (0, (i^* - 1) \cdot q_{i^*})$.[22] Consider attention-adjusted centrality vector $q' = (q'_1, \dots, q'_N)$ such that $q'_1 = q_1 + \delta$, $q'_i = q_i - \frac{\delta}{i^* - 1}$ for $i \in \{2, \dots, i^*\}$ and $q'_i = q_i = 0$ for $i \in \{i^* + 1, \dots, N\}$. Lemma 5 implies the influencer receives a higher optimal payoff when the attention-adjusted centrality vector is q'. Suppose $q_1 = N$. If $q' \neq q$, then $q'_1 < q_1$ and $q'_i \geq q_i$ for all $i \in \{2, \dots, N\}$. By Lemma 5, the influencer receives a lower optimal payoff under q'. Thus, a star network maximizes the influencer's optimal payoff.[23]

<u>Complete Network</u>:

Suppose $q_1 > 1$. By Lemma 4, $j^* < N$. Therefore, there exists a $\delta \in (0, \frac{N - j^*}{N} \cdot (q_{j^*} - q_{j^*+1}))$. Consider a vector q' such that $q'_i = q_i - \delta$ for $i \in \{1, \dots, j^*\}$ and $q'_i = q_i + \frac{j^*}{N - j^*} \cdot \delta$ for $i \in \{j^* + 1, \dots, N\}$. The vector q' satisfies $q'_1 \geq q'_2 \geq \dots \geq q'_N \geq 0$, $q'_i < q_i$ for $i \in \{1, \dots, j^*\}$ and $q'_i > q_i$ for $i \in \{j^*, \dots, N\}$. By Lemma 5A, the influencer has a lower payoff under q'. It follows that the influencer's optimal payoff is minimized when $q_1 = 1$.

Proof of Proposition 3: In a star network, each influencer allocates its entire budget to the most central agent. M_1's payoff is $b + (1 - 2b) f(1, 1)$. In a complete network, each agent has the same attention-adjusted centrality. Because agents also have a common bias, each influencer targets each agent equally. The payoff to M_1 is $b + (1 - 2b) f(\frac{1}{N}, \frac{1}{N})$.

The equilibrium payoff in a complete network is greater than the equilibrium payoff in a star network whenever $(1-2b)f(\frac{1}{N},\frac{1}{N}) > (1-2b)f(1,1)$. The proposition follows. $\square$

Appendix B: Attention-Adjusted Centrality

The attention-adjusted centrality vector q differs from other common measures of network influence in the literature, specifically eigenvector centrality. Eigenvector centrality appears in Lever (2010) and governs influencer targeting decisions in his model. However, this is due to the limited ability of the influencers' to interact with agents: Spending has a one-time effect on agents' *initial* beliefs. Thus, agents' importance is dictated entirely by their effect within the peer network. In my model, agents interact with influencers repeatedly, leading to the attention-adjusted centrality q becoming the vector of interest. Agents that are influential within the peer network do not necessarily have the same importance. However, suppose agents interact with external sources at the same rate α. As α approaches 0, q approaches the span of the eigenvector centrality.

Proposition 4. *Let w be the unit left-hand eigenvector of P associated with the eigenvalue 1. Consider any decreasing sequence $\{\alpha^{(j)}\}_{k=1}^{\infty}$ such that $\lim_{j\to\infty}\alpha^{(j)} = 0$ and the corresponding attention-adjusted centrality vectors $q^{(j)} = \alpha^{(j)}\cdot e^T(I-(1-\alpha^{(j)})P)^{-1}$. Then:*

$$\lim_{j\to\infty}\frac{q^{(j)}}{N} = w$$

Proof. Define W to be an $N\times N$ matrix where each row is equal to w. Let $\left|\left|\cdot\right|\right|_2$ denote the standard norm in $\mathbb{R}^N$. It follows that:

$$\left|\left|\frac{q^{(j)}}{N}-w\right|\right|_2 = \frac{1}{N}\left|\left|q^{(j)}-Nw\right|\right|_2 = \frac{1}{N}\left|\left|\alpha^{(j)}\cdot e^T(I-(1-\alpha^{(j)})P)^{-1}-e^TW\right|\right|_2 \tag{B.1}$$

Recognize that $\frac{W}{\alpha^{(j)}} = \sum_{t=0}^{\infty}(1-\alpha^{(j)})^tW$. This implies that (B.1) can be expressed as:

$$\frac{1}{N}\left|\left|\alpha^{(j)}e^T\sum_{t=1}^{\infty}(1-\alpha^{(j)})P^t - \alpha^{(j)}e^T\sum_{t=0}^{\infty}(1-\alpha^{(j)})^tW\right|\right|_2 \tag{B.2}$$

$$= \frac{1}{N}\left|\left|\alpha^{(j)}e^T\sum_{t=0}^{L-1}(1-\alpha^{(j)})^t(P^t-W) + \alpha^{(j)}e^T\sum_{t=L}^{\infty}(1-\alpha^{(j)})^t(P^t-W)\right|\right|_2 \text{ for any } L \tag{B.3}$$

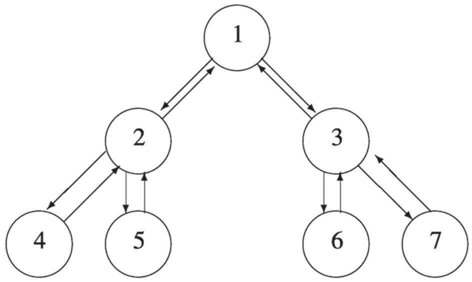
Consider any ϵ' and ϵ with $\epsilon' \in (0,\epsilon)$. Because $W = \lim_{t\to\infty}P^t$, there exists L sufficiently large such that each element of P^t is within ϵ' of each element of W for $t \geq L$. Thus:

$$\begin{aligned}(B.3) &\leq \frac{1}{N}\left|\left|\alpha^{(j)}e^T\sum_{t=0}^{L-1}(1-\alpha^{(j)})^t(P^t-W)\right|\right|_2 + \frac{1}{N}\left|\left|\alpha^{(j)}e^T\sum_{t=L}^{\infty}(1-\alpha^{(j)})^t(P^t-W)\right|\right|_2 \\ &\leq \frac{1}{N}\left|\left|\alpha^{(j)}e^T\sum_{t=0}^{L-1}(1-\alpha^{(j)})^t(P^t-W)\right|\right|_2 + (1-\alpha^{(j)})^L\epsilon' \\ &\leq \epsilon \text{ for } \alpha^{(j)} \text{ sufficiently close to } 0 \qquad \square\end{aligned}$$

The components of w are the relative impact of each agent on the others *when there is only peer-to-peer learning*. When agents are constrained to interact with their peers at the same rate, Proposition 4 implies there is a cutoff $\bar{\alpha} > 0$ such that for $\alpha < \bar{\alpha}$, the *rank ordering* of the agents according to q corresponds to that of w. For $\alpha > \bar{\alpha}$, these measures may diverge. Observe that as $\alpha \longrightarrow 1^-$ for each i, $\sum_{j=0}^{\infty}(1-\alpha)^jP^j$ puts more weight on the early terms. To illustrate, consider the following network and centrality measures for different values of α:

Example 2.

$$P = \begin{pmatrix} 0.4 & 0.3 & 0.3 & 0 & 0 & 0 & 0 \\ 0.4 & 0.4 & 0 & 0.1 & 0.1 & 0 & 0 \\ 0.4 & 0 & 0.4 & 0 & 0 & 0.1 & 0.1 \\ 0 & 0.5 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0.5 \end{pmatrix}$$



$$\alpha \to 0: \quad w = \begin{bmatrix} 0.32 & 0.24 & 0.24 & 0.05 & 0.05 & 0.05 & 0.05 \end{bmatrix}$$

$$\alpha = 0.2: \quad q = \begin{bmatrix} 8.747 & 7.73 & 7.73 & 2.70 & 2.70 & 2.70 & 2.70 \end{bmatrix}$$

$$\alpha = 0.6: \quad q = \begin{bmatrix} 1.99 & 2.12 & 2.12 & 1.36 & 1.36 & 1.36 & 1.36 \end{bmatrix}$$

As α increases, agents 2 and 3 become more central because they are separated from agents 4 – 7 by a single edge. The probability of the influencer's message being received by those outside nodes indirectly from agent 1 decreases as agents pay less attention to their peers. Agent 1 is most influential when only considering peer effects, but it influences peripheral agents **through** agents 2 and 3. As α increases, these middlemen become more important.

Example 2 highlights the tension between direct and indirect targeting. In the case of the tree, there is a bottleneck effect where the root node transmits its beliefs slowly through other agents. Thus, changes in α will have a greater effect on the centrality measure of the root. As α increases, it makes targeting peripheral agents more beneficial.